Holographic quantum matter

3. Transport

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- finite density
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U(1) conserved current J^μ dual to bulk gauge field A_a.

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Standard transport theory is Boltzmann kinetic theory, which assumes quasiparticles exist. (More on these predictions in **Lecture 4**).

Conductivity

The conductivity is formally defined as:

$$\sigma(\omega, k) = \frac{1}{\mathrm{i}\omega}\left[G^{\mathrm{R}}_{J_x J_x}(\omega, k) - G^{\mathrm{R}}_{J_x J_x}(0, k)\right]$$

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Since current J_x is dual to gauge field A_x, we should look solve bulk equations of motion subject to infalling boundary conditions:

$$A_x(r \to 0) = \mathrm{e}^{\mathrm{i}(kx - \omega t)}\left[1 + G^{\mathrm{R}}_{J_x J_x}\frac{r^{d-1}}{d-1} + \cdots\right].$$

Let's start simple: CFT in $d = 2$ (zero density). The holographic model is

$$\mathcal{L} = R - 2\Lambda - \frac{F^2}{4},$$

with background

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2 - \mathrm{d}t^2 + \mathrm{d}x^2}{r^2}, \quad A = 0.$$

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Look for solution of the form $A = A_x(r)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}x$:

$$\partial_b F^{ab} = \frac{1}{\sqrt{-g}}\partial_b \left(\sqrt{-g}g^{ac}g^{bd}F_{cd}\right) = 0$$

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The conductivity is **dimensionless**:

$$\sigma(\omega) = \frac{1}{\mathrm{i}\omega}\cdot \mathrm{i}\omega = 1.$$

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In Einstein-Maxwell theory,

[Herzog, Kovtun, Sachdev, Son; *Phys. Rev.* **D75** 085020 (2007)]

$$f = 1.$$

This is by particle-vortex duality, or $F \to *F$ duality in the bulk.

A holographic model with $f \neq 1$:

[Myers, Sachdev, Singh; *Phys. Rev.* **D83** 066017 (2011)]

$$\mathcal{L} = R - 2\Lambda - \frac{F^2}{4} + \gamma C_{abcd} F^{ab} F^{cd}.$$

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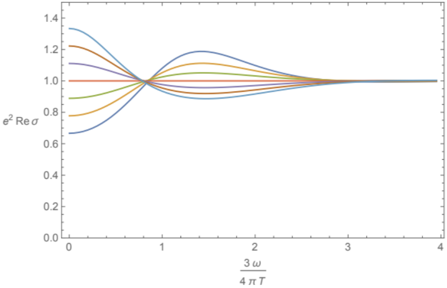
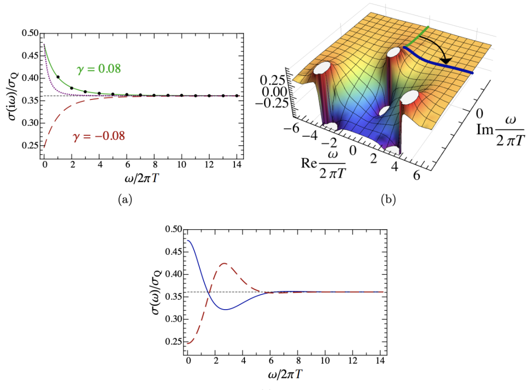
Numerical computations:



Such models can be used to analytically continue quantum Monte Carlo data from imaginary to real time!

[Witczak-Krempa, Sorensen, Sachdev; *Nature Phys.* **10** 361 (2014)]



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(c)

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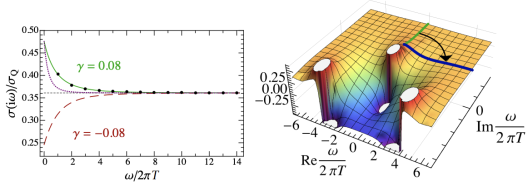

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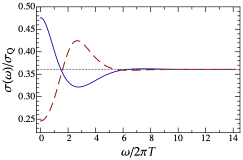

(c)

High frequency data well captured by conformal perturbation theory!

[Lucas, Podolsky, Gazit, Witczak-Krempa; *Phys. Rev. Lett.* **118** 056601 (2017)]

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Diverging momentum density π^i implies diverging current:

$$\langle J^i \rangle \approx \frac{\chi_{J^i \pi^i}}{\chi_{\pi^i \pi^i}} \langle \pi^i \rangle = \frac{\rho}{\mathcal{M}} \langle \pi^i \rangle.$$

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These conclusions do *not* rely on Lorentz, Galilean, etc. symmetry.

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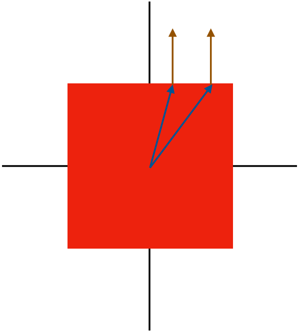
The origin of σ_{inc} is the inequivalence between current J^i and momentum π^i:



$$J_x = \int \mathrm{d}^2 p \, \frac{\partial \epsilon}{\partial p_x} f(p)$$

$$P_x = \int \mathrm{d}^2 p \, p_x f(p)$$

We predict *large* incoherent conductivity in any Fermi liquid with non-circular Fermi surface.

[Cook, Lucas; *Phys. Rev.* **B99** 235148 (2019)]

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Drude model is a standard cartoon...formally it only applies for **weak momentum relaxation**.

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Using **memory matrix methods**:

[Hartnoll, Hofman; *Phys. Rev. Lett.* **108** 241601 (2012)]

$$\mathcal{M}\cdot\Gamma = \int \frac{\mathrm{d}^d k}{(2\pi)^d} |h(k)|^2 k_x^2 \lim_{\omega\to 0} \frac{\mathrm{Im}\left[G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\omega,k)\right]}{\omega}.$$

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Can be derived *holographically*! [Lucas; *JHEP* **03** 071 (**2015**)]

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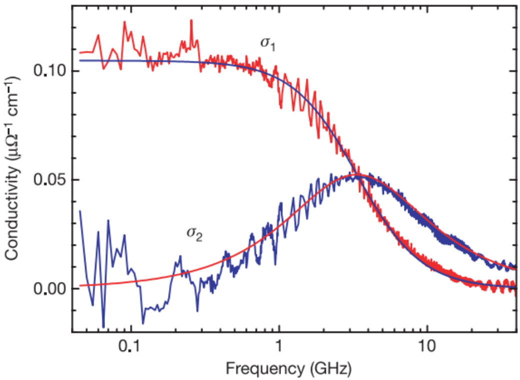
Drude in experiment on UPd_2Al_3: [Scheffler++; *Nature* **435** 1135 (2005)]



Linear axion model

Holography can implement momentum relaxation, even with spatial homogeneity of the bulk geometry! [Andrade, Withers; *JHEP* **05** 101 (**2014**)]

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with background

$$\chi^I = \frac{m}{\sqrt{2}} x^I.$$

Only $\partial_a \chi^I$ couples to equations of motion, so they stay homogeneous!

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The large m (strong momentum relaxation) limit is accessible. 👍

We can find black hole geometries at finite temperature, finite charge, and finite m!

As a simple example, consider adding $m \neq 0$ to the AdS-RN background. One finds:

$$\mathrm{d}s^2 = \frac{1}{r^2}\left[\frac{\mathrm{d}r^2}{f(r)} - f(r)\mathrm{d}t^2 + \mathrm{d}\mathbf{x}_d^2\right]$$

with

$$f(r) = 1 - \left(1 + \frac{d-1}{d}r_0^2\mu^2\right)\left(\frac{r}{r_0}\right)^{d+1} + \frac{d-1}{d}\left(\frac{r}{r_0}\right)^{2d} - \frac{m^2 r^2}{2d-2}.$$

dc conductivity can be calculated using **membrane paradigm**.

[Blake, Tong; *Phys. Rev.* **D88** 106004 (2013)]

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Rotational symmetry allows us to also turn on the following bulk perturbations:

$$\delta \tilde{g}_{tx}(r), \quad \delta \tilde{g}_{rx}(r), \quad \delta \tilde{\chi}^x(r).$$

Solve the coupled bulk equations of motion. 😩

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$$\sqrt{-g} Z F^{rx} \approx \sqrt{-g} g^{rr} g^{xx} Z(0) \partial_r A^x \to \frac{\partial_r A_x}{r^{d-2}} \sim \langle J^x \rangle.$$

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Now evaluate current at horizon $r = r_0$:

$$J \sim Z \sqrt{-g} g^{rr} g^{xx} \left(\partial_r A_x - g^{tt} \partial_r A_t \tilde{g}_{tx}\right)\Big|_{r_0}$$

Infalling boundary conditions/other EOMs:

$$g^{rr} \partial_r A_x|_{r=r_0} \sim E, \quad Z\sqrt{-g} g^{rr} g^{xx} \partial_r A_t\Big|_{r=r_0} \sim \rho, \quad g^{tt} \delta \tilde{g}_{tx}\Big|_{r=r_0} \sim \frac{\rho E}{r_0^d m^2}$$

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In these holographic models, even at $m = \infty$,

$$\sigma_{\rm dc} \geq \frac{Z(r_0)}{r_0^{d-2}}.$$

Holographic correlated systems don't have Anderson/localization transitions. This conclusion holds even for inhomogeneous black holes.

[Grozdanov, Lucas, Sachdev, Schalm; *Phys. Rev. Lett.* **115** 221601 (2015)]

It's tempting to compare:

$$\sigma_{\rm dc} = \frac{Z(r_0)}{r_0^{d-2}} + \frac{4\pi\rho^2}{r_0^d m^2} \quad \text{vs.} \quad \sigma_{\rm dc} = \sigma_{\rm inc} + \frac{\rho^2}{\mathcal{M}}\frac{1}{\Gamma - {\rm i}\omega}.$$

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However, at large m^2, discrepancies! Explicit calculation:

[Davison, Goutéraux; *JHEP* **09** 090 (**2015**)] [Blake; *JHEP* **09** 010 (**2015**)]

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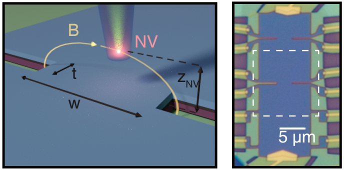
The Drude weights themselves get corrections:

[Goutéraux, Shukla; 2309.04033]

$$\sigma(\omega) = \sigma_{\mathrm{inc}} + \frac{(\rho + m^2\lambda_\rho + \cdots)^2}{(\epsilon + P)(c \cdot m^2 - \mathrm{i}\omega)}$$

It is also possible to probe *finite k* conductivity in experiments! For example, local transport probes using NV-center magnetometry:

(a)



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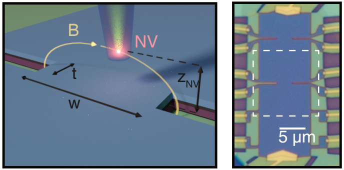
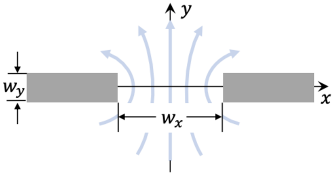


Or measure conductance of constriction of width w: heuristically,

$$G \sim \sigma\left(k \sim \frac{1}{w}\right).$$



These experiments have detected **viscous electron flow** in high-purity graphene:

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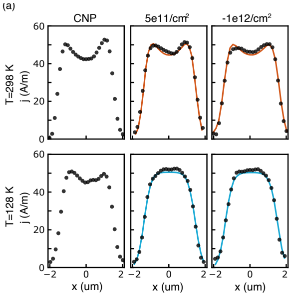
The shape of current flow detects ohmic vs. viscous flow.

[Jenkins++; *Phys. Rev. Lett.* **129** 087701 (2022)]



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Spatially-resolved transport

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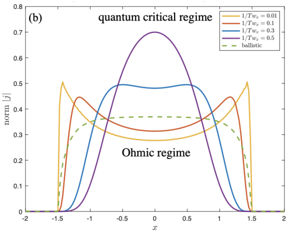
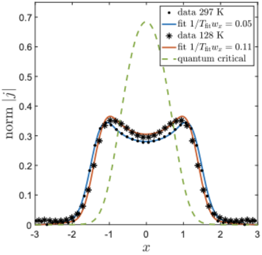
Heuristic algorithms predict space-resolved current flow profiles through constrictions, based on $\sigma(k)$.



Prediction for seeing the "quantum critical" crossover in graphene using ~ 600 nm constriction: [Huang, Lucas; *SciPost Phys.* **13** 004 (2022)]



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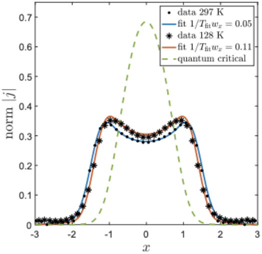

Holographic fit describes change in charge-neutral transport in graphene using one fit parameter:

$$\ell_{\rm eff} \sim \frac{C\hbar v_{\rm F}}{k_{\rm B}\,T}$$

with $C \approx 5$, compatible with optical data from graphene.

[Gallagher++; *Science* **364** 125 (2019)]

(Probe brane) holography has also been used to calculate **shot noise**, or current fluctuations in a mesoscopic device. In $d = 2$:

[Sonner, Green; *Phys. Rev. Lett.* **109** 091601 (2012)]

$$\left\langle I(t)^2 \right\rangle \sim L_{\text{width}} \times \left[T^4 + E^2 \right]^{1/4} .$$

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This has recently been used to fit to shot noise data in $YbRh_2Si_2$:

[Chen++; *Science* **382** 907 (2023)]