Holographic quantum matter

2. Phases of matter

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Recap

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conserved U(1) current J^μ	Maxwell U(1) gauge field A_a
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The bulk geometry encodes the **state** of the field theory:

finite temperature T $\to$ black hole at temp. T

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Minimally coupled theory: **Einstein-Maxwell theory:**

$$\mathcal{L} = R - 2\Lambda - \frac{F_{ab}F^{ab}}{4}$$

with Maxwell flux $F_{ab} = \partial_a A_b - \partial_b A_a$.

Add finite density by finite chemical potential:

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Consistent solution: **AdS-Reissner-Nordstrom black hole:**

[Chamblin, Emparan, Johnson, Myers; *Phys. Rev.* **D60** 064018 (1999)]

$$A_t = \mu \left(1 - \left(\frac{r}{r_0} \right)^{d-1} \right),$$

$$\mathrm{d}s^2 = \frac{1}{r^2} \left[\frac{\mathrm{d}r^2}{f(r)} - f(r)\mathrm{d}t^2 + \mathrm{d}\mathbf{x}_d^2 \right],$$

$$f(r) = 1 - \left(1 + \frac{d-1}{d} r_0^2 \mu^2 \right) \left(\frac{r}{r_0} \right)^{d+1} + \frac{d-1}{d} \left(\frac{r}{r_0} \right)^{2d}.$$

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Similar phenomenon found in SYK models.

[Sachdev; *Phys. Rev.* **X5** 041025 (2015)]

When $T \ll \mu$, geometry looks like $\mathrm{AdS}_2 \times \mathbb{R}^d$. For $r \approx r_0$, write

$$\zeta \approx \frac{r_0^2}{r_0 - r},$$

$$\mathrm{d}s^2 \approx \frac{L_2^2}{\zeta^2}\left[\frac{\mathrm{d}\zeta^2}{f_2(\zeta)} - f_2(\zeta)\mathrm{d}t^2\right] + \frac{\mathrm{d}\mathbf{x}_d^2}{r_0^2}, \quad f_2(\zeta) = 1 - (2\pi T\zeta)^2.$$

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This is a **quantum critical phase**: no parameter was tuned to realize IR criticality.

IR spectral weight

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Consider a massive scalar in the AdS-RN background:

$$\mathcal{L} = \cdots - \frac{1}{2}\partial_a\phi\partial^a\phi - \frac{m^2}{2}\phi^2.$$

In the IR $\mathrm{AdS}_2 \times \mathbb{R}^d$, we find $\phi(\zeta)\mathrm{e}^{\mathrm{i}kx-\mathrm{i}\omega t}$ with

$$\zeta^2\left[\partial_\zeta^2 + \omega^2\right]\phi = \left(\frac{1}{4} + m_{\mathrm{eff}}(k)^2\right)\phi$$

with

$$m_{\mathrm{eff}}(k)^2 = m^2L_2^2 + k^2r_0^2L_2^2.$$

We predict k-dependent critical exponents!

[Faulkner, Liu, McGreevy, Vegh; *Phys. Rev.* **D83** 125002 (2011)]

IR spectral weight

In "physics experiment", we'll measure correlation functions:

$$G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(x,t) = \Theta(t)\cdot \mathrm{i}\langle[\mathcal{O}(x,t),\mathcal{O}(0,0)]\rangle.$$

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If we accessed *only* the $\mathrm{AdS}_2 \times \mathbb{R}^d$ geometry, we'd find **spectral weight** at finite temperature:

$$\lim_{\omega\to 0}\frac{\mathrm{Im}\left[G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\omega,k)\right]_{\mathrm{IR}}}{\omega} \sim \omega^{2\Delta_k-2} \sim T^{2\Delta_k-2}.$$

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In holography, we measure correlators near the boundary $r=0$:

$$\phi(r\to 0)\sim \mathrm{e}^{\mathrm{i}(kx-\omega t)}\left[r^{d+1-\Delta}+G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\omega,k)r^{\Delta}+\cdots\right].$$

How is this not ruined by UV physics?

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We find that: [Lucas; *JHEP* **03** 071 (**2015**)]

$$\lim_{\omega\to 0}\frac{\text{Im}\left[G^{\text{R}}_{\mathcal{O}\mathcal{O}}(\omega,k)\right]}{\omega} = \frac{1}{r_0^d}\phi^{(0)}(r=r_0,k)^2.$$

IR spectral weight

$\phi^{(0)}$ itself comes from matching IR and UV:

$$\phi^{(0)}(r) \sim \begin{cases} \phi^{\mathrm{UV}}_{\mathrm{not\ norm}}(r) + c_1 \phi^{\mathrm{UV}}_{\mathrm{norm}}(r) & r \ll \mu^{-1} \\ c_2 \phi^{\mathrm{IR}}_{\mathrm{regular}}(r) & r \sim \mu^{-1} \end{cases} .$$

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Spectral weight $\mathrm{Im}\left[G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}\right]$ sensitive to IR behavior: 👍

$$\begin{aligned} \lim_{\omega \to 0} \frac{\mathrm{Im}\left[G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\omega, k)\right]}{\omega} &= \frac{1}{r_0^d} \phi^{(0)}(r = r_0, k)^2 \\ &= \frac{1}{r_0^d} \phi^{\mathrm{IR}}_{\text{regular}}(r = r_0, k)^2 . \end{aligned}$$

IR physics dominates the *dissipative spectral weight.*

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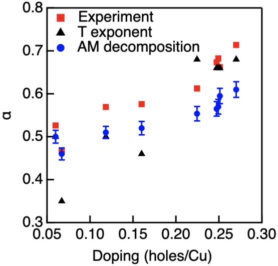
Optical conductivity in BSCO with doping-dependent exponent?

[van Heumen++; *Phys. Rev.* **B106** 054515 (2022)]



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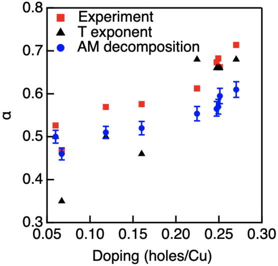

Such experiments are difficult! Real metals have competing effects, limited range of applicability, etc.

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It's debatable whether the low energy effective theory of a holographic metal should have long-lived fermions, such as

$$\mathrm{tr}\left[\Psi\Phi\right].$$

Fermi surface may also be "hidden behind the horizon".

[Sachdev; *Phys. Rev.* **D86** 126003 (2012)]

Sometimes the metallic phase is *unstable*. Suppose we have a bulk scalar with

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Onset of SSB is signaled by a (linear response) bulk mode ϕ which is unsourced and regular at the horizon.

Consider a *charged* bulk scalar:

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In the AdS-RN background,

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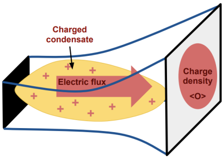
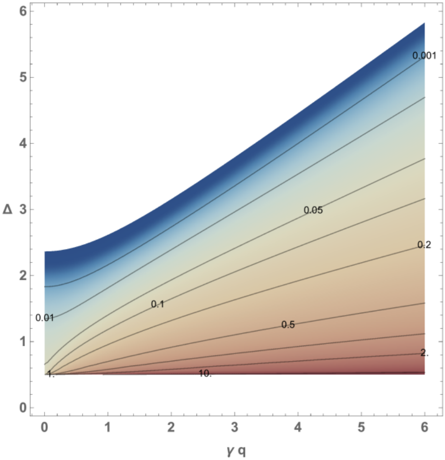
The stable solution will have a non-trivial profile $\phi(r)$ for bulk scalar:



This is the holographic **superconductor**!

Numerically solve linearized EOMs from high temperature to deduce the phase diagram for $d = 2$:

[Denef, Hartnoll; *Phys. Rev.* **D79** 126008 (2009)]



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Quantum fluctuations in the bulk ($1/N$ effects) are responsible for destroying long-range order: there's only 1 Goldstone but N^2 other degrees of freedom!

[Anninos, Hartnoll, Iqbal; *Phys. Rev.* **D82** 066008 (2010)]

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Like all cartoons, holography has limitations... 🤷‍♂️

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Address by going to **Einstein-Maxwell-dilaton gravity**:

$$\mathcal{L} = R - Z(\Phi)\frac{F^2}{4} - 2(\partial\Phi)^2 - V(\Phi).$$

The scalar dilaton Φ will be part of the background.

[Charmousis, Gouteraux, Kim, Kiritsis, Meyer; *JHEP* **11** 032 (**2010**)]

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In string theory models of holography, e^{Φ} represents the size of the compact dimensions, and one expects:

$$Z(\Phi) \sim Z_0 e^{\alpha\Phi}, \qquad V(\Phi) = -V_0 e^{\beta\Phi}.$$

A family of charged black hole solutions:

$$\begin{aligned}
\Phi(r) &\sim \log r, \\
A_t(r) &\sim r^{\theta - d - z}, \\
\mathrm{d}s^2 &\sim r^{2\theta/d} \left(\frac{\mathrm{d}r^2 + \mathrm{d}\mathbf{x}_d^2}{r^2} - \frac{\mathrm{d}t^2}{r^{2z}} \right).
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Top-down holography: truncate string theory.

Can also build *interpolating* geometries between $(z,\theta)_{\mathrm{UV}}$ and $(z,\theta)_{\mathrm{IR}}$.

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Are these more general than holography? 🤔

Universal low temperature black hole metric:

$$\mathrm{d}s^2 \sim r^{2\theta/d}\left(\frac{\mathrm{d}r^2}{r^2 f(r)} - f(r)\frac{\mathrm{d}t^2}{r^{2z}} + \frac{\mathrm{d}\mathbf{x}_d^2}{r^2}\right),$$

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Einstein-Maxwell had $z = \infty$, so $s \sim T^0$.

Scaling exponents

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$\Phi \neq 0$ (and $\theta < 0$) may be large-N artifacts, with e.g. spectra of field mass/charge.

[Karch; *JHEP* **07** 021 (**2015**)]

Non-relativistic holography has analogous dictionary. Adding scalar

$$\mathcal{L} = \mathcal{L}_{\text{EMD}} - \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}B_0 \mathrm{e}^{\beta\Phi}\psi^2,$$

the choice of B_0 determines operator dimension:

[Lucas, Sachdev, Schalm; *Phys. Rev.* **D89** 066018 (2014)]

$$\psi(r) = \underbrace{\psi_0 r^{d+z-\Delta-\theta/2}}_{\text{source}} + \underbrace{\psi_0 r^{\Delta-\theta/2}}_{\text{response}} + \cdots$$

Here the operator dimension is defined as

$$\langle \mathcal{O}(x,0)\mathcal{O}(y,0)\rangle \sim \frac{1}{|x-y|^{2\Delta}}.$$

Holography is helpful for tackling quantum field theory with random-field disorder:

$$\mathcal{L} = \mathcal{L}_{\mathrm{QFT}} - h(x)\mathcal{O}(x,t),$$

with *time-independent, quenched disorder*:

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$$\mathcal{L} \to \sum_{a=1}^{n} \mathcal{L}_{\mathrm{QFT},a} - D \sum_{a,b=1}^{n} \mathcal{O}_a(x) \int \mathrm{d}t \mathcal{O}_b(x,t).$$

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In holography, no replicas; but disordered boundary conditions:

$$\psi(r\to 0, x) = h(x)r^{d+z-\Delta-\theta/2} + \cdots.$$

Find the resulting inhomogeneous black hole! 👍

Let $[x] = -1$. Then if $[\mathcal{O}] = \Delta$,

$$[D] = d - \theta + z - 2\Delta.$$

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Harris relevant	$\nu > 0$
Harris marginal	$\nu = 0$
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Harris marginal (or relevant!) disorder could change IR fixed point.

Let's study $d = 1$, $z = 1$, $\theta = 0$, with Harris marginal disorder $\nu = 0$.

$$S = \int \mathrm{d}^3 x \sqrt{-g} \left(R + 2 - \frac{1}{2}(\partial\psi)^2 + \frac{3}{4}\psi^2 \right).$$

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Claim: there is a line of Lifshitz fixed points in the IR with

$$z_* = 1 + \frac{D}{8}, \quad \theta_* = 0.$$

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Idea: similar to QFT, naive perturbation theory suggests:

$$g_{tt} \sim -\frac{1}{r^2} - \frac{D}{4}\frac{\log r}{r^2} + \cdots$$

which should be resummed to give Lifshitz exponent z_*.

[Hartnoll, Santos; *Phys. Rev. Lett.* **112** 231601 (2014)]

However, in the Lifshitz geometry, use $m^2 = -\frac{3}{4}$ to find:

[Ganesan, Lucas; *JHEP* **06** 023 (**2020**)]

$$\Delta_* \approx \frac{3}{2} + \frac{3D}{16} > \Delta_{\text{marginal}} = \frac{3}{2} + \frac{D}{8}.$$

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A more careful analysis shows that disorder is marginally irrelevant:

[Ganesan, Lucas, Radzihovsky; *Phys. Rev.* **D105** 066016 (2022)]

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$$\frac{\mathrm{d}D(E)}{\mathrm{d}\log E} = \beta_D = D^2 \frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}} = \frac{D^2}{8}.$$

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Deviation from Lifshitz scaling visible at energy scale

$$E_{\text{IR}} = \Lambda_{\text{UV}} \mathrm{e}^{-c/D},$$

and requires **non-perturbative resummation of bulk geometry.**

Now consider weakly Harris-relevant disorder: $[D] = 2\nu$,

$$\beta_D = -2\nu D + D^2 \frac{dC_{\mathcal{O}\mathcal{O}T}}{C_{TT}}.$$

There is now a flow to a fixed point at

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Construction generalizes to charged, non-relativistic models!

[Huang, Sachdev, Lucas; *Phys. Rev. Lett.* **131** 141601 (2023)]